CLIC 3 TeV Beam size Optimization with Radiation Effects

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Abstract

Oide effect and radiation in bending magnets are reviewed aiming to include this in the optical design process to minimize the beam size. The Oide double integral is expressed in simpler terms in order to speed up calculations. Part of the Oide function is used to evaluate how prone is a quadrupole magnet to contribute to the beam size increase, concluding in larger magnets with lower gradients. Radiation in bending magnets is reviewed for linear lattices, solving the case when the dispersion is different from zero and using the result to compare with theoretical results and a tracking code. An agreement between the theory, the implemented approximation included in MAPCLASS2 and the six-dimensional radiation in PLACET has been found.

INTRODUCTION

In order to achieve higher luminosity, it is necessary to reduce the beam size to compensate the lower frequency collisions in linear accelerators compared with storage and collider rings. Due to the beam energy radiation effects should not be disregarded during the design stage of lattices. This document addresses two particular radiation phenomena: the Oide effect [1] and the radiation caused by bending magnets.

The beam size is calculated as the sum of the contributions from linear transport, non-linear aberrations and radiation effects. The calculated second momentum, σ^2, can be expressed as the sum of the second momentum of each effect. Then, the total beam size is calculated as:

$$\sigma^2 = \sigma_0^2 + \sigma_i^2 + \sigma_{rad}^2$$

The Oide effect is caused by the interaction of charged particles with the magnetic field from quadrupoles. It imposes a limit on the beam size, specially notable in the vertical plane. For the horizontal direction, is mainly affected by radiation caused by the interaction of charged particles with the magnetic field from dipoles. Both effects can be evaluated by tracking particles through lattice or by analytical approximations.

OIDE EFFECT

The Oide effect is the contribution to beam size due to radiation while particles traverse quadrupole magnets with gradually reduced momentum [1]. For the vertical plane,

$$\sigma_{rad}^2 = \frac{110}{3\sqrt{6\pi}} r_e \frac{\lambda_e}{2\pi} \gamma^5 F(\sqrt{k}L, \sqrt{k}l^*) \left(\frac{\epsilon}{\beta^*}\right)^{5/2}$$

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where $F(\sqrt{k}L, \sqrt{k}l^*)$ is

$$\int_0^{\sqrt{k}L} |\sin\phi + \sqrt{k}l^* \cos\phi|^3 \left[\int_0^{\phi} (\sin\phi' + \sqrt{k}l^* \cos\phi')^2 d\phi'\right]^2 d\phi$$

λ_e is the Compton wavelength of the electron, r_e is the electron radius, γ is the relativistic factor, ϵ is the beam emittance, β^* is the twiss parameter function at the observation point, in this case, the IP; and, k, L, and l^* are the quadrupole gradient, the quadrupole length and the focal distance (the distance to the IP).

Although the total contribution to beam size depends on lattice and beam parameters, the function F is calculated only from quadrupole parameters. For this reason, F will be used as figure of merit for a quadrupole. Aiming to accelerate the calculation of the Oide effect by computational methods, then, it will be easier to express F as a function where all integration terms have been solved.

Solving the Integrals

The inner integral over ϕ' can be solved because it has a known primitive and its result squared is always positive because all inner terms are real. The equation can then be expressed as a single integral. The term inside the absolute value is also always positive, therefore, its contribution to beam size is always positive. Considering the function $|\sin\phi + \sqrt{k}l^* \cos\phi|$, the point where the sign changes is determined by the solution of

$$\sin\phi + \sqrt{k}l^* \cos\phi = 0 \qquad \text{where} \quad k \geq 0$$

The solution is then a ϕ_n value for which $\tan\phi_n$ is negative and equal in magnitude to $\sqrt{k}l^*$. First, let's suppose that the integral limits extend over a value $\sqrt{k}L$ that does not cross any of the solutions of the previous equation, i.e., the absolute value function has not reach its first zero (e.g. 0 to $\pi/2$). The integral has an primitive, and it will be represented as

$$F(\sqrt{k}L, \sqrt{k}l^*) = \hat{F}|_0^{d_0}$$

the primitive evaluated between the limits of the integral (in this case, $d_0 = \sqrt{k}L$).

If the integration interval is extended over several zeros, then, the result extends to $d_n = \sqrt{k}l$. The change of signs in each interval is only given by the absolute value definition, then, it is simpler to add the absolute value of each contribution:

$$F(\sqrt{k}L, \sqrt{k}l^*) = |\hat{F}|_0^{z_0}| + |\hat{F}|_{z_0}^{z_1}| + |\hat{F}|_{z_1}^{z_2}| + \cdots + |\hat{F}|_{z_{n-1}}^{d_n}|$$

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If we know the primitive $\hat{F}$ and we are able to extract the zeros in the integration interval, then, it is possible to calculate the factor F without using an approximate integrator. The double integration has been simplified to a method to search for zeros and a function evaluation.

Finding Optimum F

The result was included as a python function to calculate F for different magnets. A quadrupole, can be characterized by the values K, L, l^*, the gradient, its length and its focal length. However, they are not independent. For this reason, there will be two possible approaches to the three values used for the calculation of F:

- thin lens approximation: for a defined value of K and a target focal length l^*, the length L of the quadrupole corresponds to $L = \frac{1}{Kl^*}$.
- β propagation: having the β value at the IP can be back propagated through the drift and then into the quadrupole. For a given gradient K, the length into the quad required to obtain $\alpha = 0$ is used as the quad length L plus 10%, in order to calculate F for a magnet slightly larger than minimum.

From the previous two approaches, the calculation of F was performed, see Fig. 1.

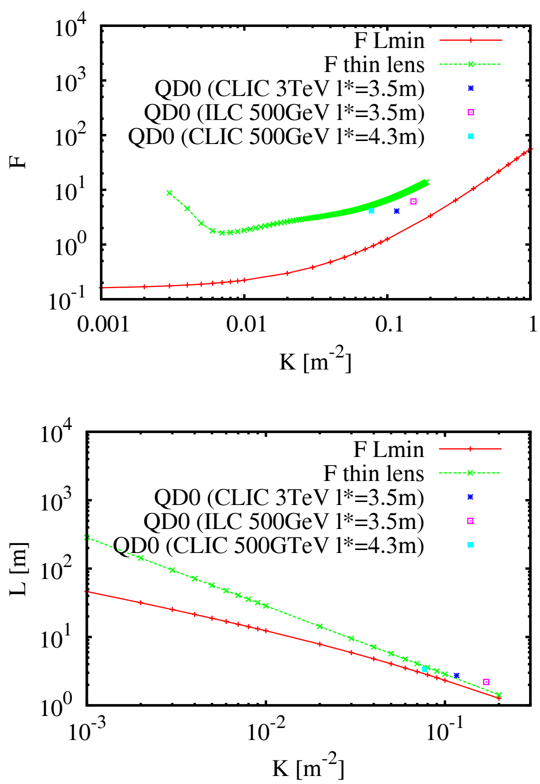

Figure 1: F as a function of magnet gradient. L is included for reference.

It is shown how the value F decreases for lower values of gradient field while preserving the focal length. The minimum length L_{min} curve gives the lowest estimate of the contribution to radiation, while the thin lens approximation is shown as an overestimation. Three additional dots have been superimposed on Fig. 1, to show the cases of current magnet QD0 in three different lattice designs: CLIC 500 GeV, ILC 500 GeV and CLIC 3 TeV.

Conclusion

It is possible to reduce the radiation contribution to beam size from the magnet by using longer quadrupoles with lower gradient. Calculations have been performed for magnets up to hundreds of meters, although unrealistic due to construction limitations, it shows that there is a limit on the optimization. Even increasing the length and decreasing constantly the gradient, it will reach a point where there is no longer improvement. In the original paper [1] a minimum $\sigma_y^* \approx 1$ mm is found.

RADIATION IN A BENDING MAGNET

Theoretical Approximation

Having a lattice whose behaviour is linear in the sense that along the accelerator the propagation can be described by the transport matrix from s_1 to s_2, then radiation can be calculated by the model described by Sands [2, 3].

$$x = \sum_{i=1}^{N(T)} \Delta x_{i,total} - x_0$$

$\Delta x_{i,total}$ is the total deviation due to the i^{th} photon radiated, x_0 is $\langle \sum_{i=1}^{N(T)} \Delta x_{i,total} \rangle$,in order to make $\langle x \rangle = 0$, and $\sigma_{rad}^2 = \langle x^2 \rangle$, N is the number of photons radiated, T is the time to cross the bending magnet. The beam size increase due to radiation will be

$$\begin{aligned} \sigma_{rad}^2 &= \langle N \rangle \langle \overline{\Delta x}^2 \rangle \\ &= C_2 \int_0^{s_p} \frac{E^5}{\rho^3} R_{16}(s, s_p)^2 ds \end{aligned}$$

where C_2 is $\frac{55}{24\sqrt{3}} \frac{r_e \hbar c}{(mc^2)^6} = 4.13 \times 10^{-11} [\mathrm{m}^2 \cdot \mathrm{GeV}^{-5}]$.

Approximation with Dispersion

The propagation of the kick generated by the radiation of a photon has two components: a betatron oscillation (rms) and a displacement from the reference orbit (mean) proportional to the dispersion.

$$\begin{aligned} \Delta x_{i,total} &= \Delta x_i + \Delta x_{i,\eta} \\ &= \frac{u}{E} \sqrt{\frac{\beta_L}{\beta_i}} [\eta_i \cos \Delta\phi_{s_i,L} + (\alpha_i \eta_i + \beta_i \eta_i') \sin \Delta\phi_{s_i,L}] \end{aligned}$$

In presence of dispersion, the contribution to beam size σ_{rad}^2 due to radiation now can be calculated as:

$$C_2 \int \frac{E^5}{\rho^3} \left\{ \sqrt{\frac{\beta_L}{\beta_s}} [\eta \cos \Delta\phi(s, L) + (\alpha\eta + \beta\eta') \sin \Delta\phi(s, L)] - \eta_L \right\}^2 ds$$

This expression has been included in MAPCLASS2 (See [4, 5]).

One Dipole

Theoretical calculation has been derived and confirmed using [6] and [7] for the case of one sector magnet and a sector magnet plus a drift. For a sector magnet, using $R_{16} = \rho(1 - \cos\theta)$

$$\sigma^2_{rad} = C_2 \int_0^\theta \frac{E^5}{\rho^3}[\rho(1-\cos(\theta-\chi))]^2 \rho d\chi$$

Including a drift in front of the bending magnet, the theoretical calculation gives:

$$\sigma^2_{rad} = C_2 \int_0^\theta E^5 \left[1 - \cos(\theta-\xi) + \frac{l}{\rho}\sin(\theta-\xi)\right]^2 d\xi$$

where, $\frac{l}{\rho} = \frac{l}{L}\theta$. Previous integrals have been calculated for E constant and will be used to normalize the results from MAPCLASS2 and PLACET (See [8]).

Before placet-0.99.02, there were two different implementations to radiation: the "default" method and the method activated with the "six_dim" flag. The former calculates radiation by segmenting the dipole in shorter pieces, what is called thin dipole approximation. The second does not make sectioning of the dipole. Their results and MAPCLASS2 results can be compared in Fig. 2 normalized to theoretical value for constant magnet length. Energy is 1500 GeV in all cases. Since placet-0.99.02 "six_dim" has become the default method due to these results.

MAPCLASS2 partially depends on MAD-X [9] twiss parameters calculation, however, common function "twiss" did not give expected analytical results for angles above 10^{-3} rad (B field of 1 T). The MAD-X function "ptc_twiss" 5 dim was used instead, giving the visible jump for angles around $5{\times}10^{-7}$ rad in Fig. 2. These angle values correspond to cases where a low number of photons is emitted and validity of the model still is on discussion.

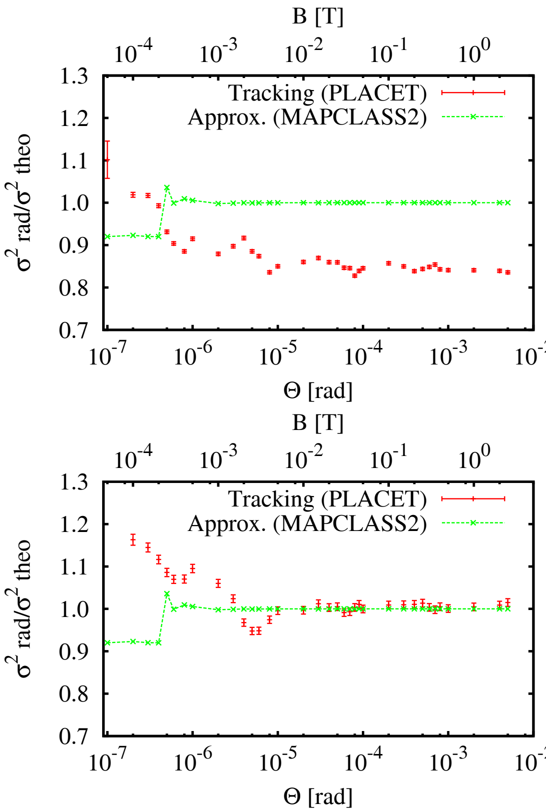

Figure 2: "Default" (up) and "-six_dim 1" (down)

ADDENDUM

This work has been developed as part of the program to perform computational optimization of lattices. It is foreseen to be used for the CLIC FFS, and can also serve for other e^+e^- optimization cases where a low number of photons per particle is under investigation.

ACKNOWLEDGEMENTS

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